



# De Sitter vacua in ghost-free massive gravity theory



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## ABSTRACT

We present a simple procedure to obtain a large class of different versions of the de Sitter solution in the ghost-free massive gravity theory via applying the Gordon ansatz. For these solutions the physical metric describes a hyperboloid in 5D Minkowski space, while the flat reference metric depends on the Stuckelberg field  $T(t, r)$  subject to  $(\partial_t T)^2 - (\partial_r T)^2 = 1$ . This equation admits infinitely many solutions, hence there are infinitely many de Sitter vacua with different physical properties. Only the simplest solution with  $T = t$  has previously been studied, as it is manifestly homogeneous and isotropic, but this solution turns out to be unstable. However, other solutions could be stable. We require the timelike isometry to be common for both metrics and this gives physically distinguished solutions since only for them the canonical Killing energy is time-independent. We conjecture that these solutions minimize the energy and are therefore stable. We also show that in some cases solutions can be homogeneous and isotropic in a non-manifest way such that their symmetries are not obvious. All of this suggests that the theory may admit physically interesting cosmologies.

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## 1. Introduction

The discovery of the ghost-free massive gravity theory by de Rham, Gabadadze, and Tolley (dRGT) [1] (see [2,3] for a review) opens up the possibility to explain the dark energy and the cosmic acceleration [4,5] by a tiny mass of the gravitons. The dRGT field equations admit the de Sitter solution with the cosmological constant mimicked by the graviton mass. This solution can describe the late time cosmic acceleration, but a special analysis is needed to decide whether its other properties are physically acceptable.

A closer look reveals that the de Sitter solution in the dRGT theory is actually not unique, and a number of its versions have been found [6–14]. A special attention was received by one particular solution whose physical and reference metrics are both of the manifestly homogeneous and isotropic Friedmann–Lemaître–Robertson–Walker (FLRW) form [10]. However, a detailed analysis reveals that this solution is unstable [15,16]. For other known solutions only the physical metric is manifestly FLRW while the reference metric looks inhomogeneous, for which reason they are considered to be less interesting [9]. All of this has reduced the interest towards the dRGT theory, the focus shifting towards its

extensions, as for example the bigravity [17–25] and other generalizations admitting FLRW solutions [26–29].

However, we would like to argue in this paper that it may be premature to abandon the dRGT theory on the basis of negative evidence obtained from just one solution, because the theory actually admits infinitely many other solutions that could be physically interesting. They all have the same physical (de Sitter) metric but different values of the reference metric depending on the Stuckelberg field  $T(t, r)$  subject to a complicated differential equation [9, 11–14]. Below we shall describe a simple way to obtain these solutions by applying the Gordon ansatz [30] and using the global embedding coordinates for the de Sitter space. The  $T$ -equation then assumes a simple form,  $(\partial_t T)^2 - (\partial_r T)^2 = 1$ , whose essentially general solution is known. The simplest solution  $T = t$  is unstable [15,16] but other solutions could be stable. One can choose  $T(t, r)$  in such a way that both metrics are invariant under the timelike isometry, which gives distinguished solutions since only for them the canonical Killing energy is time independent. We conjecture that their energy is minimal and hence these solutions are stable. We also give explicit examples where the reference metric looks inhomogeneous but shares with the physical metric the same translational and rotational isometries. Hence, solutions previously considered to be non-FLRW can actually be homogeneous and isotropic. All of this suggests that physically interesting dRGT cosmologies may exist.

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